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"Low" Energy GUTs †

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Abstract

We introduce a new approach to the subject of grand unification which allows the GUT scale to be small, $\lesssim$ 200TeV, so that it is within the reach of *conceivable* laboratory accelerated colliding beam devices. Central to the approach is a novel abstraction of the heterotic string symmetry group physics ideas to render baryon number violating effects small enough to have escaped detection to date.

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The structure of the Standard Model(SM) [1,2], in view of its success, leads naturally to the suggestion that all forces associated with the gauge interactions therein may be unified into a single gauge principle associated with a larger group $\mathcal{G}$ which contains the SM gauge group $SU(2)_L \times U(1)_Y \times SU(3)^c$ as a subgroup, where we use a standard notation for the SM gauge group. Originally introduced in the modern context in Refs. [3,4], this idea continues to be a fashionable area of investigation today, where approaches which unify the SM gauge forces with that of quantum gravity are now very much in vogue via the superstring theory [5,6] and its various low energy reductions and morphisms [6]. In what follows here, we focus only on the unification of the SM gauge forces themselves, candidates for which we call as usual GUTs, so that we leave aside any possible unification with quantum gravity until a later study [7].

We admit that a part of our motivation is the recent progress in the approaches to the Einstein-Hilbert theory for quantum gravity in Refs. [8–11] in which improved treatments of perturbation theory via resummation methods, the asymptotic safety approach, the resummed quantum gravity approach or the Hopf-algebraic Dyson-Schwinger equation renormalization theory approach, and the introduction of an underlying loop-space at Planck scales, loop quantum gravity, all support the view that the apparently bad unrenormalizable behavior of the Einstein-Hilbert theory may be cured by the dynamical interactions or modifications within the theory itself, as first anticipated by Weinberg [8]. In what follows, we explore the suggestion, which follows from such progress, that the unification of all other forces can be a separate problem from the problem of treating the apparently bad UV behavior of quantum gravity.

Our objective is to formulate GUTs so that they are accessible to very high energy colliding beam devices such as the VLHC, which has been discussed elsewhere [12] with cms energies in the 100-200TeV regime. We show in what follows that we can achieve such GUTs that satisfy the usual requirements: no anomalies, unified SM couplings, baryon stability, absence/suppression of other unwanted transitions and naturalness requirements (this may just mean N=1 susy here [13]). Here, we add the new condition that the theory will live in 4-dimensional Minkowski space. We call this our *known physical reality condition*. The most demanding requirement will be seen to be baryon stability.

To illustrate why the most difficult aspect of a GUT with a (several) hundred TeV unification scale is the issue of baryon number stability we note that the proton must be stable to $\sim 10^{29-33}$yrs, depending on the mode. Standard methods can be used to show that the natural lifetime for physics with a 100TeV scale for a dimension 6 transition in a state with the size and mass of the proton is ~ 0.01yr for example. Clearly, some new mechanism is needed to suppress the proton decay process here.

In proceeding to isolate such a mechanism, we will use what is sometimes called a radically conservative approach - we will try to rely on well-tested ideas used in a novel way. In this way we may hope to avoid moving the GUT scale to $\sim 10^{13}$TeV as it is usually done [14], or invoking hitherto unknown phenomena, such as extra dimensions [14,15], etc. We notice that the fundamental structure of a GUT theory has it organized by gauge

sector, by family sector and by Higgs sector for spontaneous symmetry breaking. We turn now to the family and gauge sectors. Let us also note that, in effecting this discussion, we present here a different realization of the basic ideas we already introduced in Ref. [16]. Only experiment can tell us which realization is used by Nature.

Specifically, the $\mathbf{10} + \bar{\mathbf{5}}$ of $SU(5)$ was advocated in Ref. [4] and shown to accommodate the SM family with a massless neutrino. With the recent advent of neutrino masses [17, 18], we must extend this fifteen dimensional representation to a sixteen dimensional representation. We choose to use the $\mathbf{16}$ of $SO(10)$ [19], as it decomposes as $\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ under an inclusion of $SU(5)$ into $SO(10)$. From the heterotic string formalism [5, 6](we view here modern string theory as an extension of quantum field theory which can be used to abstract dynamical relationships which would hold in the real world even if the string theory itself is in detail only an approximate, mathematically consistent treatment of that reality, just as the old strong interaction string theory [20] could be used to abstract properties of QCD such as Regge trajectories even before QCD was discovered) we know that in the only known and accepted unification of the SM and gravity, the gauge group $E_8 \times E_8$ is singled-out when all known dualities [6] are taken into account to relate equivalent superstring theories. A standard breakdown of this symmetry to the SM gauge group and family structure is as follows [6]:

$$\begin{aligned}
E_8 &\to SU(3) \times E_6 \to SU(3) \times SO(10) \times U'(1) \\
&\to SU(3) \times SU(5) \times U''(1) \times U'(1) \\
&\to SU(3) \times SU(3)^c \times SU(2)_L \times U(1)_Y \times U''(1) \times U'(1)
\end{aligned}$$

where the SM gauge group is now called out as $SU(3)^c \times SU(2)_L \times U(1)_Y$. It can be shown that the $\mathbf{248}$ of E_8 then splits under this breaking into $(\mathbf{8}, \mathbf{1}) + (\mathbf{1}, \mathbf{78}) + (\mathbf{3}, \mathbf{27}) + (\bar{\mathbf{3}}, \overline{\mathbf{27}})$ under $SU(3) \times E_6$ and that each $\mathbf{27}$ under E_6 contains exactly one SM family 16-plet with 11 other states that are paired with their anti-particles in helicity via real representations so that they would be expected to become massive at the GUT scale. Let us consider that we have succeeded with the heterotic string breaking scenario to get 6 families [14] under the first E_8 factor, E_{8a}, in the $E_8 \times E_8$ gauge group. They are singlets under the second $E_8 \equiv E_{8b}$. We take the first 3 families to be those with the known light leptons and the remaining 3 families to be those with the known light quarks. The quarks in the families with the known light leptons are at a scale M_{QL} that is beyond current experimental limits on new quarks; the leptons in the families with the known light quarks are at a scale M_{LL} that is beyond the current experimental limits on heavy leptons. We now repeat the same pattern of breaking for the second factor E_{8b} as well and we leave open the issue of observable families under this E_{8b}, as they may exist in principle as well. The scales M_{QL}, M_{LL} are bounded by the grand unified theory (GUT) scale M_{GUT}. This scenario stops baryon instability: the proton can not decay because the leptons to which it could transform via (leptoquark) bosons are all at too high a scale. The extra heavy quarks and leptons just introduced here may of course appear already at the LHC.

The ordinary electroweak and strong interaction gauge bosons are now an unknown mixture of the two copies of such bosons from the two $E_8's$ associated to heterotic string

theory[1]: when we break the two E_8's each to a product group $SU(3) \times E_6$ and then subsequently break each of the two E_6's to get two copies of $SU(3)^c \times SU(2)_L \times U(1)_Y$, for the initially massless gauge bosons for $SU(3)^c_i \times SU(2)_{Li} \times U(1)_{Yi} \in E_{8i}$, G^a_i, $a = 1, \cdots, 8$, $A^{i'}_i$, $i' = 1, \cdots, 3$, B_i, $i = 1, 2$, in a standard notation, we assume a further breaking at the GUT scale so that the following linear combinations are massless at the GUT scale M_{GUT} while the orthogonal linear combinations acquire masses $\mathcal{O}(M_{GUT})$ –

$$\begin{aligned} A^{i'}_f &= \sum_{i=1}^{2} \eta_{2i} A^{i'}_i \\ B_f &= \sum_{i=1}^{2} \eta_{1i} B_i. \end{aligned} \tag{1}$$

The mixing coefficients $\{\eta_{aj}\}$ satisfy

$$\sum_{i=1}^{2} \eta^2_{ai} = 1, \ a = 1, 2$$

.

For the strong interaction, we take the minimal view that the quarks in each of the families from the two E_8's are confined. We use discrete symmetry to set the two strong interaction gauge couplings to be equal at the GUT scale. This means that for the known quarks we have gluons G^a_1. Of course, experiments may ultimately force us to break the as yet unseen color group. This is straightforward to do following Ref. [21].

For the low energy EW bosons, we have some freedom in (1). We note the following values [22, 23] of the known gauge couplings at scale M_Z:

$$\begin{aligned} &\alpha_s(M_Z)|_{\overline{MS}} = 0.1184 \pm 0.0007 \\ &\alpha_W(M_Z)|_{\overline{MS}} = 0.033812 \pm 0.000021 \\ &\alpha_{EM}(M_Z)|_{\overline{MS}} = 0.00781708 \pm 0.00000098 \end{aligned} \tag{2}$$

It is well-known [25] that the factor of almost 4 between $\alpha_s(M_Z)$ and $\alpha_W(M_Z)$ and between $\alpha_W(M_Z)$ and $\alpha_{EM}(M_Z)$ when the respective unified values are 1 and 2.67 require $M_{GUT} \sim 10^{13} - 10^{12}$TeV. Here, with the use of the $\{\eta_{kj}\}$ we can absorb most of the discrepancy between the unification and observed values of the coupling ratios so that the GUT scale is not beyond current technology for accelerated colliding beam devices.

More precisely, we can set

$$\begin{aligned} \eta_{21} &\cong \frac{1}{\sqrt{2.000}} \\ \eta_{11} &\cong \frac{1}{\sqrt{3.260}} \end{aligned} \tag{3}$$

[1] If one wants to avoid any reference to superstring theory, one can just postulate our symmetry and families as needed, obviously; we leave this to the discretion of the reader.

and this will leave a "small" amount of evolution do be done between the scale M_Z and M_{GUT}.

Indeed, with the choices in (3), and the use of the one-loop beta functions [2], if we use continuity of the gauge coupling constants at mass thresholds with one such threshold at $m_H \cong 120\text{GeV}$ and a second one at $m_t = 171.2\text{GeV}$ for definiteness to illustrate our approach, then the GUT scale can be easily evaluated to be $M_{GUT} \cong 136\text{TeV}$, as advertised. For, we get,

$$b_0^{U(1)_Y} = \frac{1}{12\pi^2}\begin{cases} 4.385 & , \ M_Z \leq \mu \leq m_H \cong 120\text{GeV} \\ 4.417 & , \ m_H < \mu \leq m_t \\ 5.125 & , \ m_t < \mu \leq M_{\text{GUT}} \end{cases} \tag{4}$$

from the standard formula [2]

$$b_0^{U(1)_Y} = \frac{1}{12\pi^2}\left(\sum_j n_j\left(\frac{Y_j}{2}\right)^2\right) \tag{5}$$

where $b_0^{U(1)_Y}$ is the coefficient of g'^3 in the beta function for the $U(1)_Y$ coupling constant g' in the $SU(2)_L \times U(1)_Y$ EW theory of Glashow, Salam and Weinberg [1], n_j is the effective number of Dirac fermion degrees of freedom, i.e., a left-handed Dirac fermion counts as $\frac{1}{2}$, a complex scalar counts as $\frac{1}{4}$, and so on. Similarly, for the QCD and $SU(2)_L$ theories, we get the analogous

$$b_0^{SU(2)_L} = \frac{-1}{16\pi^2}\begin{cases} 3.708 & , \ M_Z \leq \mu \leq m_H \cong 120\text{GeV} \\ 3.667 & , \ m_H < \mu \leq m_t \\ 3.167 & , \ m_t < \mu \leq M_{\text{GUT}} \end{cases} \tag{6}$$

$$b_0^{QCD} = \frac{-1}{16\pi^2}\begin{cases} 7.667 & , \ M_Z \leq \mu \leq m_t \\ 7 & , \ m_t < \mu \leq M_{\text{GUT}} \end{cases} \tag{7}$$

from the standard formula [2]

$$b_0^{\mathcal{H}} = \frac{-1}{16\pi^2}\left(\frac{11}{3}C_2(\mathcal{H}) - \frac{4}{3}\sum_j n_j T(R_j)\right) \tag{8}$$

where $T(R_j)$ sets the normalization of the generators $\{\tau_a^{R_j}\}$ of the group $\mathcal{H}$ in the representation R_j via $\text{tr}\tau_a^{R_j}\tau_b^{R_j} = T(R_j)\delta_{ab}$ where δ_{ab} is the Kronecker delta and $C_2(\mathcal{H})$ is the quadratic Casimir invariant eigenvalue for the adjoined representaion of $\mathcal{H}$. These results (4,5,6,7,8) together with the standard one-loop solution [2]

$$g^2_{\mathcal{H}}(\mu) = \frac{g^2_{\mathcal{H}}(\mu_0)}{1 - 2b_0^{\mathcal{H}} g^2_{\mathcal{H}}(\mu_0)\ln(\mu/\mu_0)} \tag{9}$$

allow us to compute the value $M_{GUT} \cong 136$TeV for the values of η_{ij} given in (3). Here, we use standard notation that $g^2_{\mathcal{H}}(\mu)$ is the squared running coupling constant at scale μ for $\mathcal{H} = U(1)_Y,\ SU(2)_L,\ QCD \equiv SU(3)^c$.

For illustration we have chosen the value of 136TeV for the unification scale.In principle any value between the TeV scale and the Planck scale is allowed in our approach and wait for experiment to tell us what the true value is.

We sum up with the following observation, already made in Ref. [16]: instead of the traditional "desert" [4, 25] between the TeV scale and the GUT scale, we propose here a "green pasture".

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